

Tri-bimaximal Mixing from Cascades

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Abstract

We study fermion mass matrices of the cascade form which are compatible with the tri-bimaximal lepton mixing and generation mass hierarchy. The flat-cascade lepton matrices imply a parameter-independent relation among the mixing angles and mass eigenvalues. The relation has several indications that the atmospheric neutrino mixing angle is close to maximal and the other two angles have a correlation independently of neutrino mass eigenvalues. We also discuss phenomenological aspects of the cascade matrices; flavor-violating rare decays of charged leptons, thermal leptogenesis, and leptonic CP violation. Possible dynamical origins of the cascades are illustrated based on flavor symmetry and in higher-dimensional theory.

1 Introduction

Neutrino physics is one of the most important clues to seek further physics beyond the standard model (SM). The neutrino oscillation experiments are going into a new phase of precision measurements of generation mixing angles and mass squared differences. The generation mixing in the lepton sector has been found to be quite different from that in the quark sector: there are large mixings among the three-generation leptons. Various recent observations have been indicating that the experimental data of lepton mixing converges to the tri-bimaximal form [1, 2], which is given by

$$V_{\text{TB}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1.1)$$

up to complex phases of light neutrino mass eigenvalues. The current experimental data [3] of mixing angles is well approximated by V_{TB} and in turn implies a specific form of mass matrix for light neutrinos. For light Majorana-type neutrinos, the mass matrix in the flavor basis of e , μ and τ becomes

$$\begin{aligned} M_L &= V_{\text{TB}}^* \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} V_{\text{TB}}^\dagger \\ &= \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \end{aligned} \quad (1.2)$$

where $m_{1,2,3}$ are the mass eigenvalues of light neutrinos. It is found from this expression that the experimentally favored neutrino matrix is restricted to a special form in which the matrix elements are integer (inter-family related) valued. Such a suggestive form seems to indicate a hidden structure in nature beyond the SM, and a number of proposals to unravel it have been elaborated [4].

In this paper, we investigate the neutrino and charged-lepton mass matrices in the cascade form. While the cascade-form matrix has hierarchical orders of matrix elements in generation space, it can generate large lepton mixing, in particular, the tri-bimaximal mixing in the lepton sector, as will be shown in Section 3. Such a compatibility of large generation mixing with mass hierarchy is suitable for the extensions to the quark sector and grand unification. The cascade-form matrix implies a parameter-independent relation among the lepton mixing angles and mass eigenvalues, which would be tested in future neutrino oscillation experiments. In Section 4, several phenomenological aspects of the

cascade lepton matrices are also discussed, e.g. the lepton flavor-violating processes, the thermal leptogenesis, and the CP violation in neutrino oscillations. In Section 5 we present possible dynamical origins of cascades in flavor symmetric theory and in higher-dimensional spacetime. Section 6 is devoted to summarizing the results.

2 Cascade matrix

In this paper we investigate the following form of mass matrix:

$$M_{\text{cas}} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v \quad (2.1)$$

with the small parameters $|\delta| \ll |\lambda| \ll 1$. The dimension-one parameter v denotes the overall mass scale and is given by some scalar expectation value times the largest element of Yukawa matrix. There are generally $\mathcal{O}(1)$ coefficients in the matrix elements, not explicitly written in the above, and so M_{cas} is not necessarily left-right symmetric. The matrix (2.1) is called the cascade form in the view of its hierarchical structure of matrix elements (see Fig. 1). To clarify the property of cascade matrix, we will show in parallel the results of the following matrix form which has been well studied in the literature:

$$M_{\text{wat}} = \begin{pmatrix} \delta^2 & \delta\lambda & \delta \\ \delta\lambda & \lambda^2 & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad (2.2)$$

where $\mathcal{O}(1)$ coefficients have also been dropped in the matrix elements. For comparison, the generation mixings are set to be of the same order between the above two types of matrices. The mass matrix (2.2) has a more rapid stream of hierarchy flow than the cascade one (see Fig. 1) and is called here the waterfall mass matrix. The waterfall matrix is realized, for example, in the Froggatt-Nielsen model [5] with abelian flavor symmetry, where the mass terms are effectively induced from higher-dimensional operators including a scalar field ϕ whose expectation value is smaller than some cutoff scale Λ ; $\langle\phi\rangle/\Lambda \equiv \rho \ll 1$. Given a quantum charge assignment $Q_{1,2,3}$ for the three-generation fields, the matrix (2.2) is interpreted as $\delta \sim \rho^{Q_1}$, $\lambda \sim \rho^{Q_2}$, and $1 \sim \rho^{Q_3}$. Here we have taken the equal charges for left and right-handed fermions, for simplicity. The examples of dynamics for M_{cas} will be discussed in a later section by use of flavor symmetry.

The two types of matrices have the same orders of generation mixing angles θ_{12} , θ_{23} , θ_{13} , while they induce different mass eigenvalues $m_{1,2,3}$, as shown in the following table:

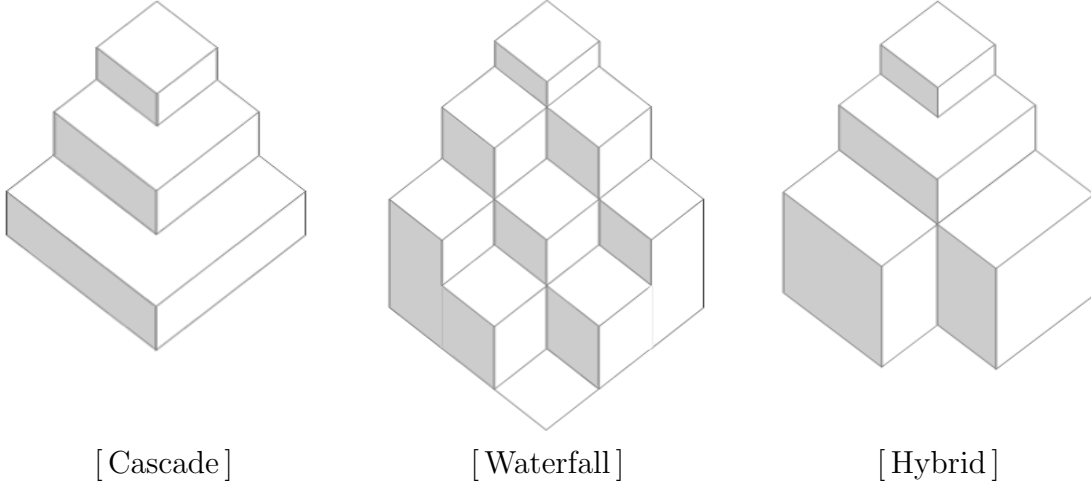


Figure 1: Hierarchy flows of matrix elements.

	M_{cas}	M_{wat}	
masses	$m_1 : m_2 : m_3 \sim \delta : \lambda : 1$	$m_1 : m_2 : m_3 \sim \delta^2 : \lambda^2 : 1$	(2.3)
mixing	$\theta_{12} \sim \delta/\lambda, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta$ $\left(\theta_{ij} \sim \frac{m_i}{m_j} \right)$	$\theta_{12} \sim \delta/\lambda, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta$ $\left(\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}} \right)$	

The relations between the eigenvalues and mixing angles are given by $\theta_{ij} \sim m_i/m_j$ for the cascade matrix and $\theta_{ij} \sim \sqrt{m_i/m_j}$ for the waterfall matrix. It is interesting here to remember the well-known relations among the quark generation mixing and the down-type quark masses m_{d_i} . The experimentally observed values of quark masses and mixing angles are roughly related as

$$\theta_{12}^q \sim \sqrt{\frac{m_{d1}}{m_{d2}}}, \quad \theta_{23}^q \sim \frac{m_{d2}}{m_{d3}}, \quad \theta_{13}^q \sim \theta_{12}^q \theta_{23}^q. \quad (2.4)$$

It is found from these expressions that θ_{12}^q is induced from a waterfall matrix, while θ_{23}^q is described by a cascade matrix. This hybrid pattern can be achieved if the cascade form is slightly modified, that is, the 1-1 matrix element is made vanishing. The hybrid cascade matrix takes the form

$$M_{\text{hyb}} = \begin{pmatrix} 0 & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v. \quad (2.5)$$

The generation mixing is set to be of the same order of the previous two matrices M_{cas} and M_{wat} . The resultant mass hierarchy and mixing angles are estimated as

	M_{hyb}	
masses	$m_1 : m_2 : m_3 \sim \delta^2/\lambda : \lambda : 1$	
mixing	$\theta_{12} \sim \delta/\lambda, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta$ $\left(\theta_{12} \sim \sqrt{\frac{m_1}{m_2}}, \quad \theta_{23} \sim \frac{m_2}{m_3}, \quad \theta_{13} \sim \frac{\sqrt{m_1 m_2}}{m_3} \right)$	(2.6)

That implies the (modified) cascade form is viable in the quark sector. Furthermore the cascade matrix is suitable for the lepton sector, as we will discuss in the following sections. Thus the cascade form matrix is expected to be embedded into grand unified theory.

3 Cascade lepton matrices

3.1 Neutrino sector

We first consider the situation that the neutrino Dirac mass matrix takes the cascade form:

$$M_N = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \delta_2 & \lambda_1 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix} v \quad (3.1)$$

with the mass parameter hierarchy $|\delta_i| \ll |\lambda_j| \ll 1$, having in mind the extension to more fundamental theory including quarks and grand unification. Throughout this paper the cascade matrix is assumed to be left-right symmetric, which is the simplest example and may be preferable to be realized. For the Majorana mass matrix of right-handed neutrinos, we have

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}. \quad (3.2)$$

It is noted that, if one would assume that M_R is also of the cascade form, the following results in this paper do not change qualitatively. This is because, as we will show, the right-handed neutrino masses are experimentally required to have larger hierarchy than the Dirac masses and therefore the generation mixing (the off-diagonal elements) in M_R becomes negligible. Accordingly we are allowed to take the diagonal form of M_R from the beginning.

The first task is to find experimental indications on the cascade neutrino matrices, referring to the current experimental data of neutrino mass eigenvalues and mixing. (The charged-lepton contribution to the lepton generation mixing will be included in the next

section.) After integrating out the heavy right-handed neutrinos [6], one obtains the Majorana mass matrix for three generations of light neutrinos in low-energy effective theory:

$$M_L = \frac{v^2}{M_1} \begin{pmatrix} \delta_1^2 & \delta_1\delta_2 & \delta_1\delta_3 \\ \delta_1\delta_2 & \delta_2^2 & \delta_2\delta_3 \\ \delta_1\delta_3 & \delta_2\delta_3 & \delta_3^2 \end{pmatrix} + \frac{v^2}{M_2} \begin{pmatrix} \delta_2^2 & \delta_2\lambda_1 & \delta_2\lambda_2 \\ \delta_2\lambda_1 & \lambda_1^2 & \lambda_1\lambda_2 \\ \delta_2\lambda_2 & \lambda_1\lambda_2 & \lambda_2^2 \end{pmatrix} + \frac{v^2}{M_3} \begin{pmatrix} \delta_3^2 & \delta_3\lambda_2 & \delta_3 \\ \delta_3\lambda_2 & \lambda_2^2 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix}. \quad (3.3)$$

Comparing this with the experimentally favored form (1.2) and taking into account the cascade hierarchy $|\delta_i| \ll |\lambda_j| \ll 1$, we are approximately lead to the following relations among the parameters:

$$\delta_1 = \delta_2 = \delta_3 \quad (\equiv \delta), \quad \lambda_1 = -\lambda_2 \quad (\equiv \lambda). \quad (3.4)$$

These are not the claims of fine tuning but should be interpreted as a first approximation for the current experimental data (remember that the tri-bimaximal generation mixing is almost at the center of the experimentally allowed region of parameter space). Such types of parameter relations have often been seen in the lepton mass models, e.g. with the vacuum alignments and non-abelian flavor symmetry which connects different generations [7]. In this paper, we study phenomenological results of the cascade lepton matrices with (3.4) as the first approximation in a suggestive form, and discuss its characteristic property in current and future particle/cosmological experiments such as for lepton flavor and CP violations. Later (in Section 5), we will present several flavor symmetry dynamics for the cascade-form matrix.

The mass eigenvalues are roughly given by $m_{\nu 1} \sim v^2/M_3$, $m_{\nu 2} \sim \delta^2 v^2/M_1$, and $m_{\nu 3} \sim \lambda^2 v^2/M_2$. These masses (and the tri-bimaximal generation mixing) are perturbed by the small quantities $m_{\nu 1}/m_{\nu 2,3}$ and δ/λ . The cascade neutrino model has the normal hierarchy of light neutrino mass spectrum, and the mass eigenvalues are explicitly given by

$$m_{\nu 1} = \frac{v^2}{6M_3}, \quad (3.5)$$

$$m_{\nu 2} = \frac{v^2}{3M_3} + \frac{3\delta^2 v^2}{M_1}, \quad (3.6)$$

$$m_{\nu 3} = \frac{v^2}{2M_3} + \frac{2\lambda^2 v^2}{M_2}, \quad (3.7)$$

including the leading-order corrections of $\mathcal{O}(m_{\nu 1})$. On the same order of perturbation evaluation, the effective neutrino mass matrix is almost diagonalized by the tri-bimaximal mixing matrix. Small deviations are evaluated at the first order in perturbation theory

and the mixing angles are determined as follows:

$$\sin^2 \theta_{12} = \left| \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \frac{m_{\nu 1}}{m_{\nu 2}} \right|^2, \quad (3.8)$$

$$\sin^2 \theta_{23} = \left| \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{m_{\nu 1}(3m_{\nu 3} - m_{\nu 2})}{m_{\nu 3}(m_{\nu 3} - m_{\nu 2})} + \frac{\delta}{3\sqrt{2}\lambda} \frac{m_{\nu 2}}{m_{\nu 3} - m_{\nu 2}} \right|^2, \quad (3.9)$$

$$\sin^2 \theta_{13} = \left| \frac{\delta}{\sqrt{2}\lambda} \frac{m_{\nu 3} - \frac{2}{3}m_{\nu 2}}{m_{\nu 3} - m_{\nu 2}} + \frac{\sqrt{2}m_{\nu 1}m_{\nu 2}}{m_{\nu 3}(m_{\nu 3} - m_{\nu 2})} \right|^2. \quad (3.10)$$

The hierarchical Dirac neutrino mass matrix of the cascade form induces the large generation mixing; the first term in (3.3) and the first relation in (3.4) means the tri-maximal mixing of three generations, and the second term in (3.3) and the second equality in (3.4) implies the bi-maximal mixing of the second and third generations where the cascade hierarchy $|\delta| \ll |\lambda|$ plays an important role. These together uniquely define the unitary mixing matrix of the tri-bimaximal form. The remaining third term in (3.3) has tiny generation mixing due to the mass hierarchy and only gives small corrections. In the waterfall model widely studied with $U(1)$ flavor symmetry, the tri-maximal and/or bi-maximal nature seems not to be simply captured, since the steepness in every point of the stream generally requires an elaborate form of right-handed neutrino mass matrix, which might be difficult to rely on some theoretical background.

As for high-energy couplings before the seesaw, the cascade hierarchy parameters are loosely bounded as

$$\left| \frac{\delta}{\lambda} \right|^2 \ll \frac{\Delta m_{21}^2}{\Delta m_{31}^2}, \quad (3.11)$$

in order for the model to be consistent with the observed generation mixing. Here $\Delta m_{21}^2 \equiv |m_{\nu 2}|^2 - |m_{\nu 1}|^2$ and $\Delta m_{31}^2 \equiv |m_{\nu 3}|^2 - |m_{\nu 1}|^2$ are the mass squared differences of light neutrinos and the current experimental data [3] at the 3 sigma level is

$$\begin{aligned} \sin^2 \theta_{12} &= 0.32_{-0.06}^{+0.08}, & \Delta m_{21}^2 &= 7.6_{-0.5}^{+0.7} \times 10^{-5} \text{ eV}^2, \\ \sin^2 \theta_{23} &= 0.50_{-0.16}^{+0.17}, & |\Delta m_{31}^2| &= 2.4_{-0.4}^{+0.4} \times 10^{-3} \text{ eV}^2, \\ \sin^2 \theta_{13} &< 0.050. \end{aligned} \quad (3.12)$$

Then the bound (3.11) means that the ratio between the two cascade falls is $|\delta|/|\lambda| \ll 0.16 - 0.20$. There is no experimental upper bound on $|\lambda|$, and the cascade could have a mild hierarchy.

As for the right-handed neutrino Majorana masses, they are estimated from the experimental data of neutrino oscillations. Given the normal hierarchy of light neutrino mass spectrum, the right-handed neutrino masses become

$$|M_1| \simeq 3.4 \times 10^{11} |\delta|^2 \left(\frac{v}{\text{GeV}} \right)^2 \text{ GeV}, \quad (3.13)$$

$$|M_2| \simeq 4.0 \times 10^{10} |\lambda|^2 \left(\frac{v}{\text{GeV}} \right)^2 \text{ GeV}, \quad (3.14)$$

$$|M_3| \simeq 1.9 \times 10^{12} \left(\frac{v}{\text{GeV}} \right)^2 \text{ GeV}, \quad (3.15)$$

with the best fit values of the experimental data (and no complex phase parameters assumed). The first two generation masses, M_1 and M_2 , are determined independently of the tri-bimaximal generation mixing. The third-generation mass M_3 does not have theoretical and/or experimental upper bound, and the limit $|M_3| \rightarrow \infty$ means that the lightest eigenvalue $m_{\nu 1}$ vanishes and the tri-bimaximal generation mixing is achieved with a large cascade hierarchy. On the other hand, M_3 has a lower bound which is given by the maximal deviations of $\sin^2 \theta_{12}$ and Δm_{21}^2 from their best fit values, i.e. $|M_3| \geq 3.8 \times 10^{11} (v/\text{GeV})^2 \text{ GeV}$. We thus find the right-handed neutrinos generally have the mass hierarchy $|M_1| < |M_2| \ll |M_3|$, while the largest light neutrino mass $m_{\nu 3}$ is given by the M_2 effect.

It is seen from the above discussion that there are four combinations of independent parameters while the five observed quantities exist in the solar and atmospheric neutrino oscillations. Therefore one parameter-independent relation among the observables is found (the corrections from the charged-lepton sector will be evaluated in the next section). That is explicitly written down in the leading order of $r \equiv (\Delta m_{21}^2 / \Delta m_{31}^2)^{1/2}$:

$$\frac{1}{9} \left(\sin^2 \theta_{23} - \frac{1}{2} \right) - \frac{r}{4} \left(\sin^2 \theta_{12} - \frac{1}{3} \right) - \frac{\sqrt{2} r}{27} \sin \theta_{13} = 0, \quad (3.16)$$

where we have taken the parameters as real valued. The relation (3.16) is interpreted in two ways. First, the solar neutrino angle θ_{12} has a correlation with θ_{13} . Such behavior is given independently of the detail of light neutrino mass spectrum. Figure 2 represents a typical numerical calculation of θ_{12} and θ_{13} . It can be seen from the figure that $(\theta_{12}, \theta_{13})$ is concentrated near a thin curve implied by the relation (3.16). Secondly, the relation implies that the atmospheric neutrino angle θ_{23} is near the maximal value $\pi/4$. We show in Fig. 3 the numerical evaluation of the atmospheric neutrino angle. In both these figures, the tri-bimaximal mixing is found to be realized around the central region of parameter space.

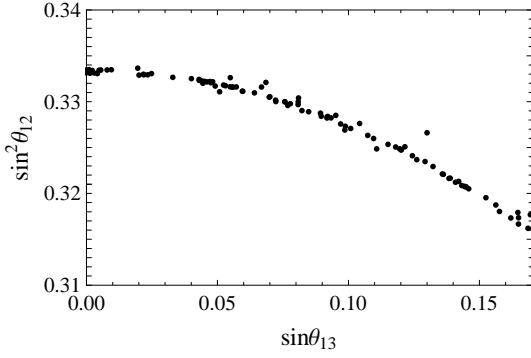


Figure 2: Typical prediction for $(\theta_{12}, \theta_{13})$ from the cascade matrices.

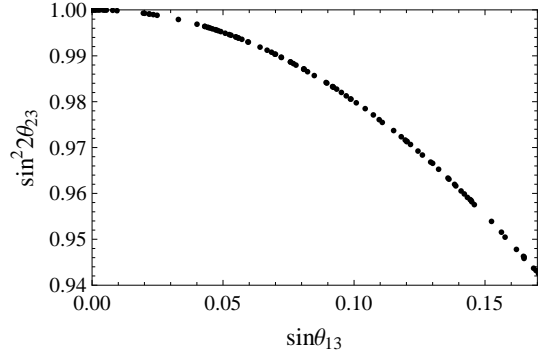


Figure 3: Near maximal θ_{23} from the cascade matrices.

Finally we comment on other similar types of scenarios for neutrino masses. First, for the asymmetric form of cascade, there is a solution which makes the seesaw-induced mass matrix consistent with (1.2). In this solution, the 3-2 element in the Dirac mass M_N is on the same order of the 3-3 element, like the so-called lopsided matrix [8]. The inverted mass hierarchy of light neutrinos is also viable for asymmetric cascades. Second, the cascade form of the neutrino Dirac mass matrix is known to be preferred from the viewpoint of parameter fine-tuning [9]. The third scenario with a different principle is the sequential dominance model [10]. In this approach, the first law of M_N is assumed to have a vanishing element to realize the tri-bimaximal generation mixing in the lepton sector. The choice of the vanishing element depends on the order of right-handed neutrino masses. The sequential dominance model has the parameter relations (3.4) in a different basis, in other words, the mass hierarchy in M_R is not necessarily sequential. As for the (symmetric) cascade matrix discussed in this paper, the hierarchical structure, $|\delta_i| \ll |\lambda_j| \ll 1$, plays important roles for realizing the tri-bimaximal mixing and neutrino mass eigenvalues. Further we are motivated to explore the cascade form (3.1) for the extensions to the quark sector and also to grand unified theory which connects the matrix forms of quarks and leptons. The symmetric and hierarchical cascade also has a peculiar dynamical origin, as will be discussed in a later section [e.g. see (5.4)].

3.2 Charged-lepton sector

As mentioned in the introduction, in this paper we explore the possibility that the mass matrix of charged leptons also has the cascade form. This is motivated, for example, by

a high-energy completion of the present framework with left-right gauge symmetry or a more fundamental principle such as grand unification. In this subsection, we study the corrections from the charged-lepton sector to the lepton generation mixing angles.

Now the charged-lepton mass matrix takes the form:

$$M_E = \begin{pmatrix} \delta_1^e & \delta_2^e & \delta_3^e \\ \delta_2^e & \lambda_1^e & \lambda_2^e \\ \delta_3^e & \lambda_2^e & 1 \end{pmatrix} v_e, \quad (3.17)$$

where $\delta_1^e \sim \delta_2^e \sim \delta_3^e \sim \mathcal{O}(\delta^e)$ and $\lambda_1^e \sim \lambda_2^e \sim \mathcal{O}(\lambda^e)$. Unlike the neutrino sector, the magnitudes of cascade hierarchy can be evaluated from the experimentally observed values of charged-lepton masses and given by

$$|\lambda^e| \simeq \frac{|m_\mu|}{|m_\tau|} \simeq 6 \times 10^{-2}, \quad (3.18)$$

$$|\delta^e| \simeq \frac{|m_e|}{|m_\tau|} \simeq 3 \times 10^{-4}. \quad (3.19)$$

The generation mixing is expressed in terms of the cascade hierarchy parameters, as shown in Table (2.3). Therefore the corrections from the charged-lepton sector are found to be generally small and the total lepton mixing angles are given at the first order of perturbation

$$\sin^2 \theta_{12} = \left| \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \frac{m_{\nu 1}}{m_{\nu 2}} - \frac{1}{\sqrt{3}} \frac{m_e}{m_\mu} \right|^2, \quad (3.20)$$

$$\sin^2 \theta_{23} = \left| \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{m_{\nu 1}(3m_{\nu 3} - m_{\nu 2})}{m_{\nu 3}(m_{\nu 3} - m_{\nu 2})} + \frac{\delta}{3\sqrt{2}\lambda} \frac{m_{\nu 2}}{m_{\nu 3} - m_{\nu 2}} - \frac{1}{\sqrt{2}} \frac{m_\mu}{m_\tau} \right|^2, \quad (3.21)$$

$$\sin^2 \theta_{13} = \left| \frac{\delta}{\sqrt{2}\lambda} \frac{m_{\nu 3} - \frac{2}{3}m_{\nu 2}}{m_{\nu 3} - m_{\nu 2}} + \frac{\sqrt{2}m_{\nu 1}m_{\nu 2}}{m_{\nu 3}(m_{\nu 3} - m_{\nu 2})} + \frac{1}{\sqrt{2}} \frac{m_e}{m_\mu} \right|^2, \quad (3.22)$$

From these expressions, one can see the effects of charged-lepton cascades. The solar neutrino mixing is little (less than 1%) affected and the tri-bimaximal solar angle ($\sin^2 \theta_{12} \simeq 1/3$) still holds. As for the atmospheric neutrino mixing, in the right-handed side of Eq. (3.21), the charged-lepton effect often gives the dominant correction ($\sim 6\%$) to the tri-bimaximal atmospheric angle ($\sin^2 \theta_{23} \simeq 1/2$). Finally, the reactor neutrino mixing sometimes receives a comparable effect relative to the neutrino sector result. However its magnitude is of negligible order ($\lesssim 1\%$) and the tri-bimaximal reactor angle ($\sin^2 \theta_{13} \simeq 0$) is not modified too much by the charged-lepton sector. Since the hierarchy of the charged-lepton cascade is expressed by the observables as (3.18) and (3.19), a

parameter-independent relation still holds including the charged-lepton correction:

$$\frac{1}{9}\left(\sin^2\theta_{23}-\frac{1}{2}-\frac{m_\mu}{m_\tau}\right)-\frac{r}{4}\left(\sin^2\theta_{12}-\frac{1}{3}\right)-\frac{\sqrt{2}r}{27}\sin\theta_{13}=0, \quad (3.23)$$

in the first order approximation.

4 Related phenomenology

As we have shown, the cascade form of lepton mass matrices is well fitted to the observed masses and mixing angles, and in particular, yields the tri-bimaximal generation mixing from hierarchical mass matrix structure. The non-trivial generation mixing in Yukawa matrices generally provides rich flavor phenomenology other than fermion masses. In this section, we investigate characteristic phenomenology induced by the cascade-form matrix: the lepton flavor violation in supersymmetric extension of the theory, the baryon asymmetry of the Universe via thermal leptogenesis, and the CP violation in neutrino oscillations.

4.1 Flavor violation

First we estimate the branching ratios of flavor-violating rare decays of charged leptons. In non-supersymmetric theory, the lepton flavor violation (LFV) is suppressed and generally negligible because the only source of low-energy LFV is the light neutrino masses and is very small relative to the electroweak scale. On the other hand, the supersymmetric (SUSY) theory generally predicts sizable magnitudes of LFV amplitudes since additional sources of LFV come from mass parameters of superparticles (scalar leptons). This type of flavor-violating vertices are radiatively generated depending on the form of lepton mass matrices. In the following, we estimate the branching ratios of the rare decay processes $\ell_i \rightarrow \ell_j \gamma$ for the cascade lepton matrices.

We consider as a simple and conservative situation that soft SUSY-breaking masses of scalar leptons are universal at some boundary scale Λ . Then their off-diagonal matrix elements are generated by radiative corrections from the Dirac Yukawa couplings of neutrinos [11]. The one-loop renormalization group evolution induces the left-handed scalar lepton masses which are approximately given by

$$(m_\ell^2)_{ij} \sim \frac{1}{8\pi^2 v^2} (3m_0^2 + |a_0|^2) \sum_k (M_N^\dagger)_{ik} (M_N)_{kj} \ln\left(\frac{|M_k|}{\Lambda}\right), \quad (\text{for } i \neq j) \quad (4.1)$$

where m_0 and a_0 denote the universal SUSY-breaking mass and three-point coupling of scalar superpartners given at the boundary scale Λ . The magnitude of these off-diagonal elements depend on the form of Dirac neutrino mass matrix M_N and the scale of right-handed Majorana masses M_i . The expression (4.1) means that the leading-order effects generally include large (i.e. the third-generation) Yukawa couplings.

The branching ratio of the $\ell_i \rightarrow \ell_j \gamma$ process is given by the loop diagrams including the vertex $(m_\ell^2)_{ij}$ in the mass insertion approximation. The result is roughly estimated as

$$\text{Br}(\ell_i \rightarrow \ell_j \gamma) \sim \frac{3\alpha}{2\pi} \frac{|(m_\ell^2)_{ij}|^2 M_W^4}{m_{\text{SUSY}}^8} \tan^2 \beta. \quad (4.2)$$

Here α and $\tan \beta$ are the fine structure constant and the ratio of two Higgs expectation values in supersymmetric SM, respectively. In the denominator, m_{SUSY} denotes a typical mass scale of superparticles circulating in the loops. In what follows, we set $m_0 = a_0 = m_{\text{SUSY}}$. Thus the branching ratios are given in the table below:

	Cascade	Waterfall
$\text{Br}(\mu \rightarrow e \gamma)$	$C \frac{m_1 m_2}{m_3^2} ^2 [\ln(\frac{ M_2 }{\Lambda})]^2$	$C \frac{m_1 m_2}{m_3^2} [\ln(\frac{ M_3 }{\Lambda})]^2$
$\text{Br}(\tau \rightarrow e \gamma)$	$C \frac{m_1}{m_3} ^2 [\ln(\frac{ M_3 }{\Lambda})]^2$	$C \frac{m_1}{m_3} [\ln(\frac{ M_3 }{\Lambda})]^2$
$\text{Br}(\tau \rightarrow \mu \gamma)$	$C \frac{m_2}{m_3} ^2 [\ln(\frac{ M_3 }{\Lambda})]^2$	$C \frac{m_2}{m_3} [\ln(\frac{ M_3 }{\Lambda})]^2$

(4.3)

We have taken into account the charged-lepton corrections but these are found to be negligible in the evaluation of LFV. The Dirac mass eigenvalues $m_{1,2,3}$ are obtained by diagonalizing M_N . The common factor C is given by $C \simeq 10^{-5} B$ where B is determined model dependently by superparticle mass spectrum and Higgs expectation values: $B \equiv (M_W/m_{\text{SUSY}})^4 \tan^2 \beta$. For comparison, we have listed in the table the results of waterfall-form mass matrix. In particular, we obtain the following relations for the cascade mass matrix:

$$\frac{\text{Br}(\mu \rightarrow e \gamma)}{\text{Br}(\tau \rightarrow \mu \gamma)} \simeq 2 |\lambda|^2 \sin^2 \theta_{13} \left[\frac{\ln(|M_2|/\Lambda)}{\ln(|M_3|/\Lambda)} \right]^2, \quad (4.4)$$

$$\frac{\text{Br}(\tau \rightarrow e \gamma)}{\text{Br}(\tau \rightarrow \mu \gamma)} \simeq 2 \sin^2 \theta_{13}, \quad (4.5)$$

where the reactor angle θ_{13} is given by the neutrino sector contribution.* The dominant

*The sequential dominance model, mentioned in Section 3, presents somewhat different phenomenological predictions. For instance, $\text{Br}(\mu \rightarrow e \gamma)/\text{Br}(\tau \rightarrow \mu \gamma) \propto \sin^2(2\sqrt{2}\theta_{13})$ when M_3 is the heaviest right-handed neutrino mass. Further, since the sequential dominance model assumes that the charged-lepton generation mixing is similar to that of quarks in the sense that the mixing is dominated by the 1-2 mixing, the reactor angle θ_{13} is determined by the correction from the charged-lepton sector. These facts are compared with the predictions from the cascade mass matrix: the ratio given in (4.4) depends on the hierarchy of neutrino mass cascade, and also the charged-lepton correction to θ_{13} is found to be small.

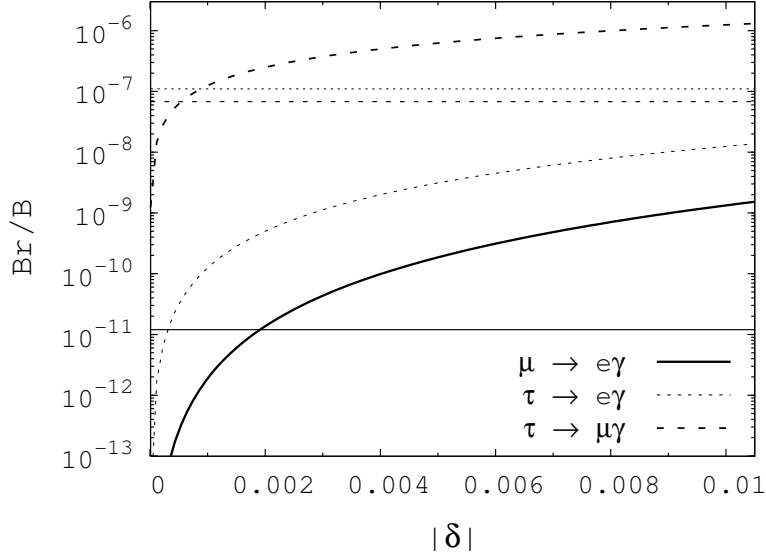


Figure 4: Typical predictions for lepton flavor violation in the cascade model. The solid, dotted, and dashed lines denote the branching ratios (over the common B factor defined in the text) of $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$, and $\tau \rightarrow \mu\gamma$, respectively. The cascade hierarchy is fixed $|\delta| = |\lambda|^2$ in the figure. The corresponding horizontal lines mean the current experimental upper bounds.

contribution to $\text{Br}(\mu \rightarrow e\gamma)$ comes from the second-generation effect in the cascade model, while all the other branching ratios depend on M_3 due to the large Yukawa coupling of the third generation. It is found from the above table that, for fixed mass eigenvalues, all the LFV processes in the cascade model are more suppressed than the waterfall model. The suppression is enough, even when $\tan\beta$ is large or the superparticle mass scale m_{SUSY} is around the electroweak scale. For example, if $|\delta| = |\lambda|^2 = 10^{-4}$, typical Majorana masses given in (3.13)-(3.15) read

$$|M_1| \sim 10^8 \text{ GeV}, \quad |M_2| \sim 10^{11} \text{ GeV}, \quad |M_3| \sim 10^{16} \text{ GeV}, \quad (4.6)$$

and the branching ratios then become

$$\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-15} B, \quad \text{Br}(\tau \rightarrow e\gamma) \sim 10^{-12} B, \quad \text{Br}(\tau \rightarrow \mu\gamma) \sim 10^{-8} B, \quad (4.7)$$

$$B = \left(\frac{M_W}{m_{\text{SUSY}}} \right)^4 \tan^2 \beta.$$

These results are compared with the current experimental upper bounds at the 90% confidence level [12]: $\text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$, $\text{Br}(\tau \rightarrow e\gamma) < 1.1 \times 10^{-7}$, and

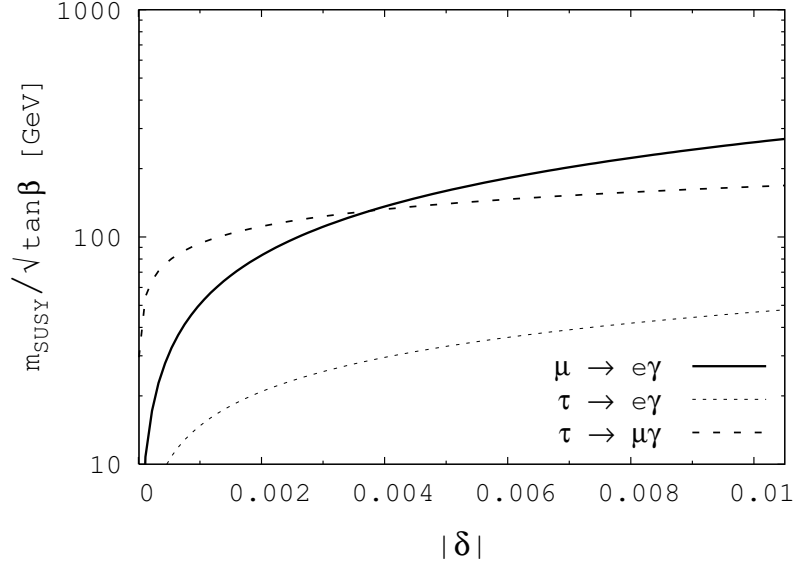


Figure 5: Typical lower bounds of superparticle mass scale which come from the LFV processes, $\mu \rightarrow e\gamma$ (solid line), $\tau \rightarrow e\gamma$ (dotted line), and $\tau \rightarrow \mu\gamma$ (dashed line). The cascade hierarchy is fixed as $|\delta| = |\lambda|^2$ in the figure.

$\text{Br}(\tau \rightarrow \mu\gamma) < 6.8 \times 10^{-8}$. The first two predictions are far below the experimental limits. On the other hand, the $\tau \rightarrow \mu\gamma$ decay is marginal to the present bound and would be observed in future LFV searches with relatively light superparticle spectrum. The branching ratios increase as the cascade factors δ and λ , and larger values of these factors lead to observable effects as shown in Fig. 4. This fact in turn constrains the mass scale of superparticles. In Fig. 5, we show the lower bound of SUSY scale m_{SUSY} for a typical hierarchy in the cascade neutrino matrix ($|\delta| = |\lambda|^2$). The figure shows that, for $|\delta| \gtrsim 3 \times 10^{-3}$ ($\lesssim 3 \times 10^{-3}$), the experimental limit from the $\mu \rightarrow e\gamma$ ($\tau \rightarrow \mu\gamma$) decay imposes the most severe constraint on the SUSY-breaking scale, while the $\tau \rightarrow e\gamma$ decay rate is too small to be detected. For larger hierarchy of the cascade, $|\delta| = |\lambda|^n$ ($n > 2$), the $\mu \rightarrow e\gamma$ decay is more suppressed but the $\tau \rightarrow \mu\gamma$ process is not. Therefore the lower bound on m_{SUSY} is weakened and becomes insignificant for larger values of $|\lambda| \gtrsim 5 \times 10^{-2}$.

4.2 CP violation

Next let us study CP-violating phenomenology, in particular, examine whether the thermal leptogenesis [13] works in the cascade model. The CP-asymmetry parameter in the decay

of the right-handed neutrino R_i is defined as

$$\varepsilon_i = \frac{\sum_j \Gamma(R_i \rightarrow L_j H) - \sum_j \Gamma(R_i \rightarrow L_j^c H^\dagger)}{\sum_j \Gamma(R_i \rightarrow L_j H) + \sum_j \Gamma(R_i \rightarrow L_j^c H^\dagger)}. \quad (4.8)$$

As seen in the previous section, the cascade model has hierarchical mass eigenvalues of right-handed neutrinos. In this case, neglecting thermal corrections, an approximate formula for ε_1 at low temperature (but reasonably accurate even at higher temperatures) is given by [14]

$$\varepsilon_1 = \frac{1}{8\pi} \sum_{j \neq 1} \frac{\text{Im}(A_{j1})^2}{|A_{11}|} F(r_j) \quad (4.9)$$

in the basis that the right-handed Majorana mass matrix is diagonalized (with real positive eigenvalues). The mass ratios of right-handed neutrinos are denoted by $r_j \equiv |M_j/M_1|^2$. The hermite matrix A is defined as $A \equiv (DM_N M_N^\dagger D^\dagger)/v^2$ where D is the diagonal phase matrix which makes the eigenvalues M_i real and positive. The loop function F is determined by evaluating the Feynman diagrams for the R_1 decay;

$$F(x) = \begin{cases} \sqrt{x} \left[\frac{2-x}{1-x} - (1+x) \ln \left(1 + \frac{1}{x} \right) \right] & \text{(SM)} \\ \sqrt{x} \left[\frac{2}{1-x} - \ln \left(1 + \frac{1}{x} \right) \right] & \text{(SUSY SM)} \end{cases} \quad (4.10)$$

Note that the loop function factor $F(r_j)$ behaves as $1/r_j^{1/2}$ for large mass hierarchy, i.e. $r_j \gg 1$. The relevant quantities for ε_1 are listed in the table below:

	A_{11}	$ A_{12} $	$ A_{13} $	M_1/M_3	M_2/M_3
Cascade	$3 \delta ^2$	$ \delta ^2$	$ \delta $	$\mathcal{O}(\delta^2)$	$\mathcal{O}(\lambda^2)$
Waterfall	$ \delta ^2$	$ \delta\lambda $	$ \delta $	$\mathcal{O}(\delta^3\lambda)$	$\mathcal{O}(\delta\lambda^3)$

(4.11)

It is found from the definition of ε_1 (or the matrix A) that the charged-lepton effect, i.e. the left-handed field rotation, does not change the CP asymmetry and need not be included. The generation mixing of Dirac neutrinos is set to be of similar order between the two types of matrices and so the hierarchy flow in the right-handed Majorana mass matrix becomes more rapid in the waterfall model. This fact leads to the result that the cascade model generally predicts larger cosmological CP asymmetry than the waterfall model. However notice that A_{12} in the cascade model is not a naive expectation $\mathcal{O}(\delta\lambda)$ but a suppressed value $\mathcal{O}(\delta^2)$. This is because of a cancellation caused by the relative sign, $\lambda_1 = -\lambda_2$, which is suggested by the current neutrino experimental data [see the third term

in the mass matrix (1.2)]. Consequently, the effect of the second generation often becomes sub-leading in the cascade model, as will be seen in the following. On the other hand, it is found from the above table that in the waterfall model the second-generation effect is dominant. The hierarchy factor dependence of the CP asymmetry is roughly estimated by dropping numerical factors as $\varepsilon_1 \sim (A_{13}^2/A_{11})(M_1/M_3) \sim \mathcal{O}(\delta^2)$ in the cascade model and $\varepsilon_1 \sim (A_{12}^2/A_{11})(M_1/M_2) \sim \mathcal{O}(\delta^2)$ in the waterfall model. Therefore the cascade hierarchy is found to generally induce similar or sometimes larger baryon asymmetry compared with the Froggatt-Nielsen like hierarchy.

From the general formula (4.9), we obtain the asymmetry parameter for the SM with the cascade mass matrix;

$$\varepsilon_1 \simeq \frac{-1}{16\pi|\delta|^2} \left[|\delta|^4 \sin(\theta_2 - \theta_1) \left| \frac{M_1}{M_2} \right| + \text{Im}[\delta^2 e^{i(\theta_3 - \theta_1)}] \left| \frac{M_1}{M_3} \right| \right], \quad (4.12)$$

where $\theta_i = \arg(M_i)$. In supersymmetric extensions, the result becomes twice that of the SM because the loop function F differs by a factor 2 when the right-handed neutrino masses are hierarchical; $|M_{2,3}| \gg |M_1|$. Moreover the decay of the superpartner of R_1 also generates roughly the same size of asymmetry as (4.12) due to the presence of supersymmetry. Given the mass hierarchy of right-handed neutrinos (3.13)-(3.15) (with the best-fit values of neutrino oscillation parameters), the ratio of the first and second terms in (4.12) is found to be $\sim 50 \times |\delta^2|/|\lambda^2|$, and hence the second term is dominant unless M_3 is huge or δ has a particular value so that $\arg(\delta^2) = \theta_1 - \theta_3$. We here define the resultant CP asymmetry η_{CP} as the ratio of the lepton asymmetry and the photon number density n_γ . This is parameterized as

$$\eta_{\text{CP}} = \frac{135 \zeta(3)}{4\pi^4} \frac{\kappa s}{g_*} \frac{\varepsilon_1}{n_\gamma}, \quad (4.13)$$

where s is the entropy density and g_* is the effective number of degrees of freedom in thermal equilibrium; $s = 7.04 n_\gamma$ in the present epoch and $g_* = 106.75$ (228.75) for the SM (for the minimal SUSY SM). The numerical factor in (4.13) denotes the equilibrium R_1 number density relative to the entropy density. As mentioned above, in supersymmetric theory the scalar neutrino decay roughly doubles the result (4.13).

The efficiency factor κ is obtained by numerically solving the Boltzmann equations and is a function of two parameters: the heavy mode mass M_1 and the effective light neutrino mass $m_{\text{eff}} \equiv |(M_N^\dagger M_N)_{11}/M_1|$. In particular, the efficiency is known to depend only on m_{eff} when $|M_1| \ll 10^{14}$ GeV, which is realized in the cascade model [see (3.13)]. That

leads to an approximate formula [15]:

$$\kappa^{-1} \simeq \frac{3.3 \times 10^{-3} \text{ eV}}{m_{\text{eff}}} + \left(\frac{m_{\text{eff}}}{0.55 \times 10^{-3} \text{ eV}} \right)^{1.16}, \quad (4.14)$$

with vanishing initial R_1 population. We have shown in the previous section that the cascade neutrino mass matrix leads to $m_{\text{eff}} = |\hat{m}_2| \simeq \sqrt{\Delta m_{21}^2} \sim 10^{-2} \text{ eV}$, and therefore the second term in (4.14) becomes dominant. The baryon number asymmetry η_B is transferred via spharelon interactions as $\eta_B = -\frac{28}{79}\eta_{\text{CP}}$ in the SM and $\eta_B = -\frac{8}{23}\eta_{\text{CP}}$ in the minimal SUSY SM. Combining the above result and the mass parameters calculated previously, we obtain the baryon asymmetry of the Universe in the cascade model [taking account only of the leading (second) term in (4.12)]:

$$\eta_B \leq 8.4 \times 10^{-6} |\delta|^2 \sin \theta_B, \quad (4.15)$$

where $\theta_B = 2 \arg(\delta) + \theta_3 - \theta_1$. An almost similar size of η_B is obtained for the SUSY SM, considering the fact that the washout effect is two times stronger because of the additional decay channel to superpartners. The current experimental data at 95% confidence level shows that $\eta_B = (4.7 - 6.5) \times 10^{-10}$ from the big bang nucleosynthesis result [12] and $\eta_B = (5.6 - 6.5) \times 10^{-10}$ from the WMAP 3-year mean result in the standard Λ cold dark matter scenario [16]. Then it is found that the magnitude of neutrino cascade hierarchy,

$$|\delta| \gtrsim (7.5 - 8.8) \times 10^{-3}, \quad (4.16)$$

is consistent with the baryon asymmetry of the Universe with $\mathcal{O}(1)$ complex phases of mass parameters. In fact, for a larger value of the cascade hierarchy, the first term in (4.12) becomes effective and must be taken into account in the analysis. Consequently the above bound of $|\delta|$ is modified by an $\mathcal{O}(1)$ factor. In Fig. 6, we plot the full numerical evaluation of phenomenological consequences of the cascade lepton matrices. In this figure, the cascade hierarchy parameters are around $|\delta| = |\lambda|^2 \sim \mathcal{O}(10^{-2})$. It is found that, for this type of hierarchy, the $\mu \rightarrow e\gamma$ rare decay process implies a lower bound on the superparticle mass scale to be larger than about $450\sqrt{\tan\beta} \text{ GeV}$, which is a bit heavier than scalar leptons in typical minimal supergravity scenarios. Note that, for a larger hierarchy of the cascade, $|\delta| = |\lambda|^n$ ($n > 2$), the only modification is more suppression of the first term in (4.12) and the above result is affected little.

The asymmetry η_B becomes tiny in the limit of a vanishing $m_{\nu 1}$ or equivalently a huge M_3 . In other words, an upper bound on M_3 , for example $|M_3| < M_{\text{GUT}}$, leads to a restricted prediction of the baryon asymmetry. The maximal value of asymmetry

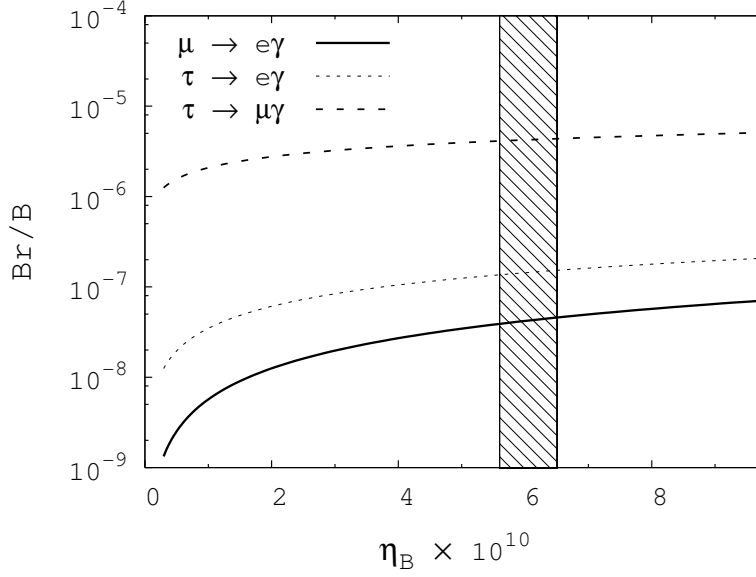


Figure 6: Phenomenology of the cascade lepton matrices: the flavor violation and the baryon asymmetry. The cascade hierarchy parameters are around $|\delta| = |\lambda|^2 \sim \mathcal{O}(10^{-2})$ in the figure. A relative complex phase between the second and third-generation effects in (4.12) is set to be constructive. The vertical shadow band is allowed by the cosmological observations at the 95% CL.

shown in (4.15) is obtained in the case of a smaller solar angle and a smaller Δm_{21}^2 within the range of experimental bounds. The corresponding lower bound (4.16) means that the Yukawa hierarchy in the neutrino sector is a bit smaller than in the charged-lepton sector ($|\delta| > |\delta^e| \sim m_e/m_\tau$). Such a hierarchy factor may be reduced, for example, by taking a larger value of $|m_{\nu 1}/m_{\nu 2}|$ with suitable complex phases. Another reasonable possibility is to consider different initial population of the right-handed neutrino R_1 at high temperature. More abundance of initial R_1 makes the efficiency factor larger. For example, if we choose the cascade factor δ as the same order of δ^e , then $|M_1| \sim 10^8$ GeV and the efficiency factor κ is enhanced by 1-3 orders of magnitude, depending on the initial R_1 abundance. This behavior makes η_B enhanced and reduces δ by 1 or 2 orders of magnitude, which is consistent with $|\delta| \lesssim |\delta^e|$.

Another important CP-violating phenomenon is the Dirac-type CP violation in neutrino oscillations, which could be observed in future long baseline experiments. The effect of the Dirac CP phase is expressed in terms of the quantity J_{CP} which is invariant under the rephasing and relabeling of fermion fields [17]. From the analysis of the generation

mixing matrix in the previous section, we find that J_{CP} from the cascade lepton mass matrices is given by

$$J_{\text{CP}} = \frac{1}{6} \text{Im} \left[\frac{\delta}{\lambda} \cdot \frac{m_{\nu 3} - \frac{2}{3}m_{\nu 2}}{m_{\nu 3} - m_{\nu 2}} + \frac{2m_{\nu 1}m_{\nu 2}}{m_{\nu 3}(m_{\nu 3} - m_{\nu 2})} + \frac{m_e}{m_\mu} \right], \quad (4.17)$$

in the leading approximation. The maximum value of J_{CP} is related to the LFV branching ratios as $\text{Br}(\tau \rightarrow e\gamma)/\text{Br}(\tau \rightarrow \mu\gamma) = (6J_{\text{CP}}^{\text{max}})^2$. Further, from the cosmological analysis, the contribution of the first term in (4.17) is found to be dominant. Thus the CP violation in neutrino oscillations is approximately described by the phase of cascade hierarchy factor; $\arg(\delta/\lambda)$. As shown hereinbefore, the cascade form of lepton mass matrices leads to characteristic and correlated behaviors for flavor physics and cosmology. That deserves to be investigated in more detail and examined in future particle experiments.

5 Illustrative toy models

In this section we show that the cascade form of mass (Yukawa) matrix has possible dynamical origins in high-energy regime. The cascade contains two step hierarchies of the orders of δ and λ . The former factor is concerned with the first generation and the latter with the second one. Further, as argued above, the neutrino experimental data would suggest that the coefficients of effective mass (Yukawa) operators are correlated to each other. These non-trivial properties imply some non-trivial implements introduced in fundamental theory beyond the standard model.

5.1 Flavor symmetry

The first example is to introduce an abelian flavor symmetry. The standard model and its extensions contain three-generation left and right-handed fermions, L_i and R_i ($i = 1, 2, 3$), and the Higgs field which has a non-vanishing vacuum expectation value v . In addition to these fundamental fields, here three gauge-singlet scalars ϕ_j ($j = 1, 2, 3$) are also included. We write down the model in a supersymmetric way using L , R , ϕ as corresponding superfields, but a non-SUSY theory is easy to construct with an additional symmetry which reflects the holomorphicity of superpotential terms. The quantum number assignment of $U(1)$ flavor symmetry is determined in the following way:

	L_1	L_2	L_3	R_1	R_2	R_3	ϕ_1	ϕ_2	ϕ_3
$U(1)$	$2m + 1$	1	0	$2m + 1$	1	0	$-2m - 3$	-2	-1

(5.1)

where m is an arbitrary positive integer. We have taken the matter $U(1)$ charges as symmetric ones ($Q_{L_i} = Q_{R_i}$) and the third-generation fields have zero charges ($Q_{L_3, R_3} = 0$). The latter fact just defines the overall scale of induced mass terms, which scale can be easily reduced by a universal shifting of all charges so that $Q_{L_3}, Q_{R_3} > 0$.

The effective mass terms come from the operators which are consistent with the flavor symmetry and are generally higher dimensional suppressed by the cutoff scale Λ , at which the operators are effectively generated by high-scale dynamics. The induced Dirac mass operators in the superpotential $W = R_i(M_D)_{ij}L_j$ are now given by

$$M_D = \begin{pmatrix} \frac{\phi_1 \phi_2^{m-1} \phi_3}{\Lambda^{m+1}} & \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2^m \phi_3}{\Lambda^{m+1}} \\ \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2}{\Lambda} & \frac{\phi_3}{\Lambda} \\ \frac{\phi_2^m \phi_3}{\Lambda^{m+1}} & \frac{\phi_3}{\Lambda} & 1 \end{pmatrix} v. \quad (5.2)$$

It would be expected that these scalar fields develop the same magnitude of expectation values $\langle \phi_1 \rangle \simeq \langle \phi_2 \rangle \simeq \langle \phi_3 \rangle \equiv \lambda \Lambda$, e.g. governed by a single sector dynamics, and as a result the mass matrix becomes

$$M_D \simeq \begin{pmatrix} \lambda^{m+1} & \lambda^{m+1} & \lambda^{m+1} \\ \lambda^{m+1} & \lambda & \lambda \\ \lambda^{m+1} & \lambda & 1 \end{pmatrix} v. \quad (5.3)$$

This is the cascade-form matrix with the hierarchy $\delta \simeq \lambda^{m+1}$ (m is an arbitrary positive integer). In fact, the quantum number assignment (5.1) is shown to be unique, up to an overall rescaling, in the case that one flavor symmetry and three gauge-singlet fields generate the cascade.

For the neutrino sector, the Majorana mass matrix of right-handed neutrinos was taken to be flavor diagonal in the previous analysis. That is realized, e.g. by introducing several scalars which transform non-trivially under additional symmetry (the lepton number or some discrete symmetry). It is however noted that, as we mentioned before, the right-handed Majorana mass matrix can also be of the cascade form, which is derived in a similar way to the above.

5.2 Extra dimensions

The second example is an extension of the SM involving the extra spacetime beyond our four dimensions and the non-abelian discrete flavor symmetry as its heritage.

An interesting key to realize the cascade form is the observation that the cascade is split into three layers (see also Fig.1):

$$M_{\text{cas}} \propto \begin{pmatrix} \delta & \delta & \delta \\ \delta & \delta & \delta \\ \delta & \delta & \delta \end{pmatrix} + \begin{pmatrix} & \lambda & \lambda \\ & \lambda & \lambda \\ & & \end{pmatrix} + \begin{pmatrix} & & \\ & & \\ & & 1 \end{pmatrix}. \quad (5.4)$$

The first and second terms indicate the existence of non-abelian flavor symmetry which leads to generation-correlated values of matrix elements. The first two terms also contain the suppression factors relative to the last term. The dynamical origin of suppression is here traced to the dilution of existence probabilities in the extra spatial dimensions.

Let us consider a six-dimensional theory on the flat gravitational background. The extra two-dimensional space is compactified on the torus T^2 with the radii R_5 and R_6 . In addition, the theory is assumed to have the Z_3 invariance which acts as the $2\pi/3$ rotation on T^2 . That implies the torus is the diamond ($R_5 = R_6 \equiv R$) with an interior angle $2\pi/3$. The torus is further divided by Z_3 and results in the orbifold T^2/Z_3 . The orbifold has three Z_3 fixed points: $P_1 = (0, 2\pi R/\sqrt{3})$, $P_2 = (\pi R, \pi R/\sqrt{3})$, and $P_3 = (0, 0)$. The assertion of the equivalence of three fixed points may lead to the existence of permutation S_3 flavor symmetry in the low-energy effective theory of this setup [18].

We briefly show a schematic picture of field configuration in the extra dimensions. The three-generation left and right-handed fermions are assumed to be generation-separately localized on the three fixed points $P_{1,2,3}$ of the orbifold. As for the bosonic sector of the theory, we show a simple example that the electroweak Higgs field H comes from a six-dimensional scalar, and further three types of gauge-singlet scalars are arranged, ϕ_1 in the bulk, ϕ_2 on a line, and ϕ_3 on a fixed point: the latter fixed point means P_3 corresponding to the third generation and the line connects two fixed points P_2 and P_3 on which the second and third-generation fermions reside.

The six-dimensional scalar ϕ_1 couples to all the three-generation fermions. If the effective theory has independent flavor symmetry for left and right-handed fermions,[†] the operators involving ϕ_1 induce the lowest layer of the cascade [the first term in the cascade matrix (5.4)] with the universal coefficient. The scalar ϕ_2 , which extends into the fifth dimension, is separate from the first-generation fields due to the locality in the extra dimensions. That results in producing the middle layer of the cascade. The coefficients of ϕ_2 operators may be controlled by a subgroup of flavor symmetry. Similarly, the four-dimensional scalar ϕ_3 couples to the third generation on the same fixed point, and hence

[†]For details of permutation flavor symmetry, see for example Ref. [19].

induces the third term in the cascade matrix (5.4). As for the hierarchy (the relative heights of the cascade layers), it has an interesting dynamical origin in the present framework: it is determined by the volume of extra-dimensional space. That is, the scalar fields $\phi_{1,2,3}$, which generate effective Yukawa operators, have different dimensionality and then provide different volume suppression factors for mass matrix elements. In the above example, the relative hierarchies are given by $\delta \simeq 1/\Lambda R$ and $\lambda \simeq 1/\sqrt{\Lambda R}$ where Λ is the cutoff scale of the theory ($\gg 1/R$). It is possible to have a larger hierarchy, $\delta \sim \lambda^n$ ($n > 2$), if the scalar ϕ_1 extends to more higher-dimensional spacetime.

Finally we comment on the Majorana mass term for right-handed neutrinos. It can be obtained in a similar way to the above by introducing extra scalar fields with different dimensionality. If these scalars have the lepton number one, the mass hierarchy in the right-handed Majorana matrix is the square of that in the Dirac one. This fact realizes in a dynamical way the result (3.13)-(3.15) with the central values of experimental data.

The higher-dimensional framework has a variety of possible field configurations in different classes of extra-dimensional space, each of which has an individual low-energy prediction. Other types of configurations are then constructed to realize the cascade-form matrix: a six-dimensional theory compactified on T^2/Z_3 with three-generation fermions being localized on three different lines, a seven-dimensional theory compactified on a torus or octahedron with three generations extending to different directions of extra three spatial dimensions, etc. The scheme given in this subsection is just for illustration and an explicit construction of a realistic complete theory is left for future study.

6 Summary

In this paper we have investigated the phenomenology of cascade mass matrices in the neutrino and charged-lepton sectors. Implementing the seesaw mechanism, an approximate tri-bimaximal generation mixing is found to be induced from hierarchical lepton mass matrices. The flat-cascade lepton matrices, which are well fitted to the experimental data, imply one parameter-independent relation among the observables, the generation mixing angles and mass eigenvalues. The relation means the correlated values of mixing angles, that behavior will be tested in future neutrino experiments.

We have discussed several phenomenological aspects of the neutrino and charged-lepton mass matrices in the flat-cascade form. The first is the flavor-violating rare decay of charged leptons in supersymmetric standard models. While the branching ratios are sup-

pressed due to small flavor mixing with fixed mass eigenvalues, several decay modes give observable effects and in turn impose the lower bound on the supersymmetry-breaking scale. The second is the CP-violating phenomena in neutrino oscillations and cosmology. The latter means the baryon asymmetry of the Universe via the leptogenesis produced in the cascade model. The predictions of these quantities are found to be correlated and make the cascade-form mass matrix testable in near future.

We have also illustrated several dynamical frameworks for realizing the cascade-form matrix. The dynamics involves the existence of flavor symmetry and/or extra spatial dimensions. Along the lines presented here, the construction of realistic model including the quark sector and the investigation of induced phenomenology will be the next important tasks to probe the existence of cascades in nature.

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